International Research Training Group IRTG 2657
Computational Mechanics Techniques in High Dimensions

An efficient space-time adaptive phase-field fracture solver with model order reduction and DIC/DVC

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Full Order PFF Problem

The variational problem based on the Euler-Lagrange equations for the phase-field fracture problem, appearing as a variational inequality, is given by the following:
For some given initial value φ^0 and for the incremental steps t_n with $n = 1, \dots, N$, find $(\mathbf{u}^n, \varphi^n) \in \mathcal{V} \times \mathcal{K}^n$ such that for all

$$(\boldsymbol{\psi}^{\mathbf{u}}, \psi^{\varphi}) \in \mathcal{V} \times (\mathcal{K}^n \cap L^\infty(\Omega))$$

it holds

$$(g(\varphi^n)\boldsymbol{\sigma}(\mathbf{u}^n), \varepsilon(\boldsymbol{\psi}^{\mathbf{u}})) + ((\varphi^n)^2 p^n, \operatorname{div} \boldsymbol{\psi}^{\mathbf{u}}) = 0,$$

and

$$\begin{aligned}(1-\kappa)(\varphi^n \boldsymbol{\sigma}(\mathbf{u}^n) : \varepsilon(\mathbf{u}^n), \psi^\varphi - \varphi^n) + 2(\varphi^n p^n \operatorname{div} \mathbf{u}^n, \psi^\varphi - \varphi^n) \\ + G_C \left(\frac{1}{\ell}(1-\varphi^n, \psi^\varphi - \varphi^n) + \ell(\nabla \varphi^n, \nabla(\psi^\varphi - \varphi^n)) \right) \geq 0,\end{aligned}$$

where the function spaces are defined by

$$\mathcal{V} = H_0^1(\Omega), \quad \mathcal{K}^n = \{\psi \in H^1 \,|\, \psi - \varphi^{n-1} \leq 0 \quad \text{a.e. in } \Omega\}.$$

The fully monolithic PFF problem yields two challenges:

- High degree of nonlinearity in the fully monolithic system
- Variational inequality appearing due to irreversibility constraints from the groundlying Griffith model

Numerical Solver

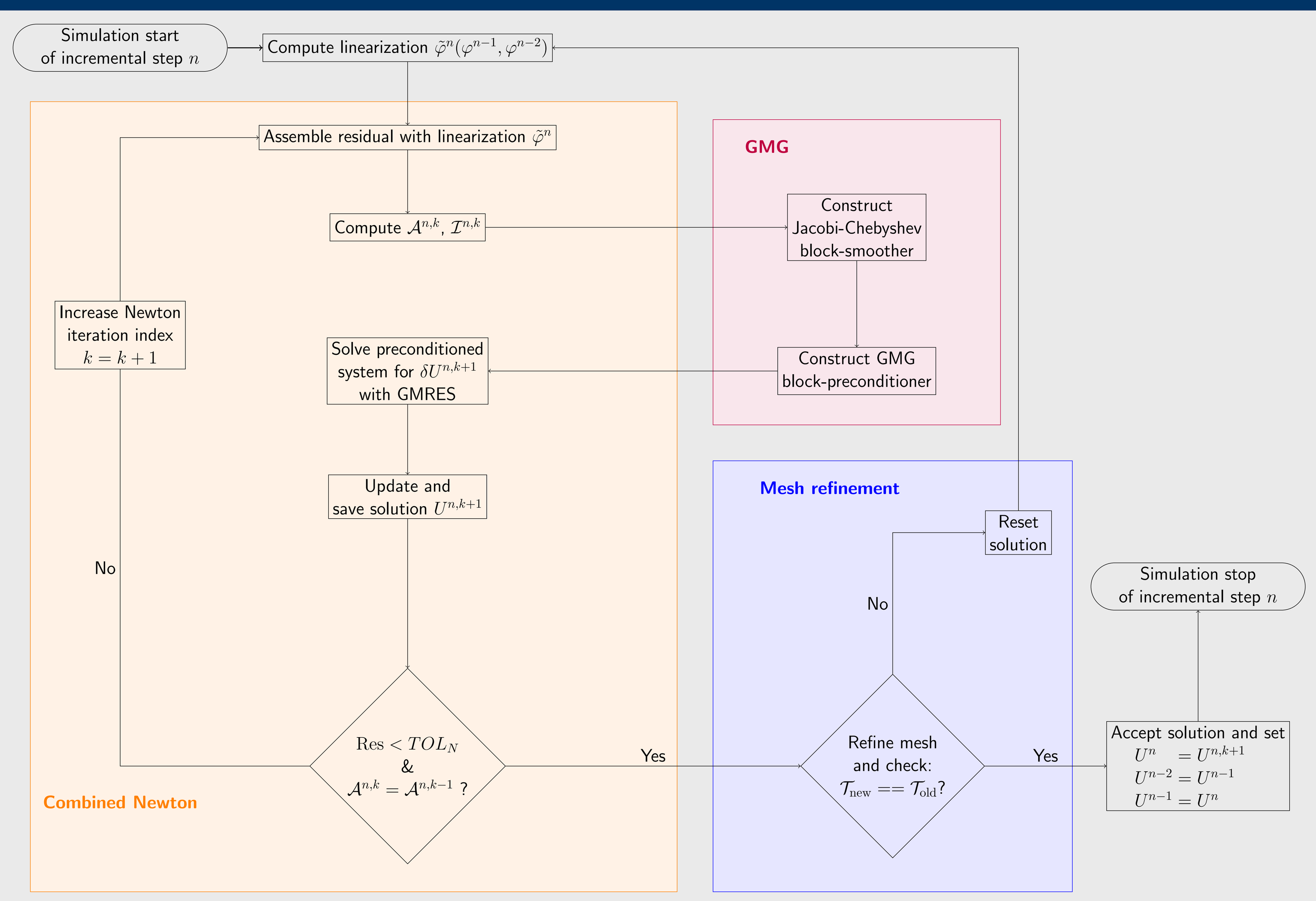


Figure: Flowchart of the space-adaptive, matrix-free, numerical solver for the full order PFF model.

Numerical Results

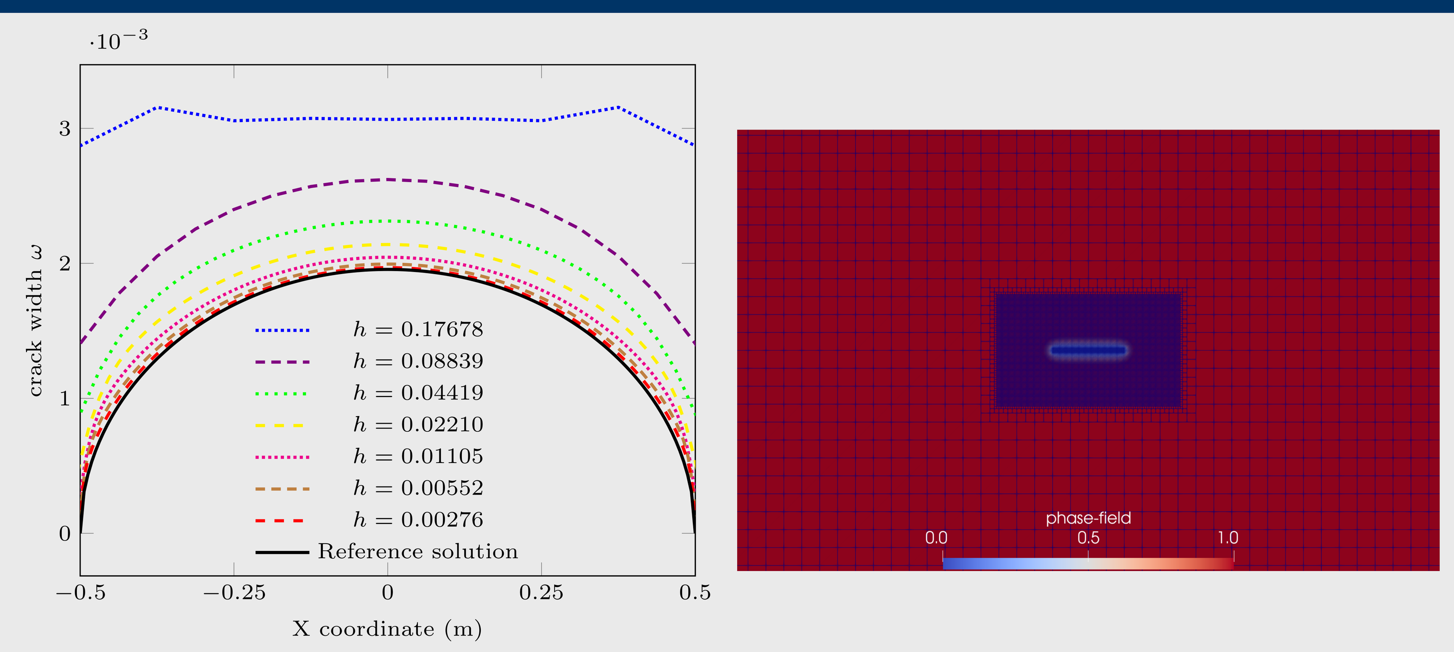
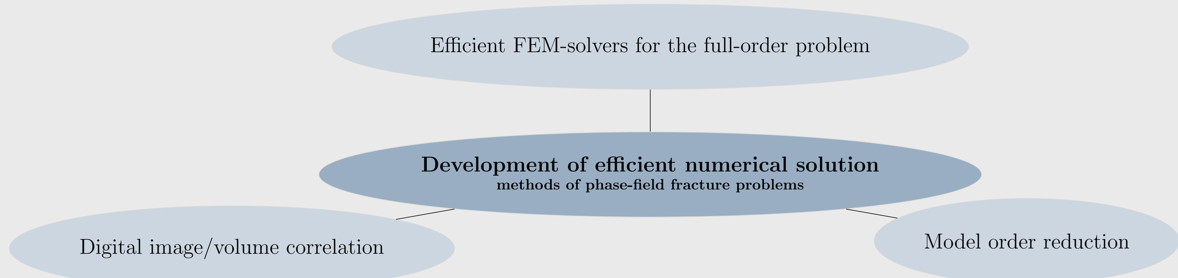


Figure: Visualisation of a phase-field solution (right) and a plot of the crack opening displacement for different diagonal element diameters.

h	# Cells	# DoFs	peak lin. iter.
0.0440	11, 728	36, 081	7
0.0220	26, 428	80, 625	7
0.0110	83, 464	252, 609	7
0.0055	308, 116	928, 305	8
0.0027	1, 199, 776	3, 606, 753	8
0.0013	4, 752, 556	14, 272, 017	8
0.0007	18, 935, 992	56, 836, 161	8

Table: Linear iterations of the Sneddon 2d benchmark test for different refinement levels. The average number of combined Newton iterations per timestep is in between $4-6$. The solver is parallelized with MPI and implemented with deal.II.

Project Overview



Publications and ongoing Work

Publications:

- Study on Γ-convergence relations in numerical solutions for PFF [2]
- Modified Newton-Active set method for PFF [3]
- Development of a Matrix-Free Multigrid preconditioner for efficient numerical solution for PFF [5]
- Extraction of advanced material parameters from FEM-solutions based on Williams Series approach [4]

Ongoing work:

- Publication of the developed full-order FEM-solver for PFF (currently in a private repository)
- Integrated DIC for parameter identification off PFF FEM-solutions with application to the Carpiuc-benchmark [1]
- A Matrix-Free Multigrid preconditioner for efficient numerical solution for fluid-filled phase-field fracture problems (collab. with S. Lee, Florida, USA)
- A global coarsening multigrid preconditioner for PFF (collab. with T. Heister, Clemson, USA and M. Steinbach, PhoenixD, Hannover)

Research Plans for 2025-2027

- 2026: Integrated DIC methods for parameter identification based on FEM solutions
- 2026-2027: Matrix-free goal-oriented error estimation for space-time adaptivity based on prior work by Viktor Kosin [6]
- 2027: Investigation of MOR techniques for PFF problems
- 2027: Finishing the thesis

IRTG Environment

- Interdisciplinary collaborations with LMPS (ENS Paris-Saclay)
  → More expertise regarding mechanics
- Collaboration with ENS offers special oppertunities:
  → Performing a tension test experiment
- Internal collaborations on discussions on HPC, matrix-free methods, PFF and DIC/DVC with Ramakrishnan Chandran and Viktor Kosin
- Talks at Conferences:
  - ENUMATH 2025
  - Invitation to Young Researchers' Minisymposium on "Robust and Reliable Solution Techniques for Multiphysics in Porous Media" at GAMM 2026

References

[1] Andreea Carpiuc, Martin Poncelet, Julien Réthoré, and Stéphane Roux. "CARPIUC benchmark overview: crack advance, reorientation, propagation and initiation under complex loadings". In: *Advanced Modeling and Simulation in Engineering Sciences* 5.1 (2018), p. 24.

[2] **Leon Kolditz** and Katrin Mang. "On the relation of Gamma-convergence parameters for pressure-driven quasi-static phase-field fracture". In: *Examples and Counterexamples* 2 (2022), p. 100047.

[3] **Leon Kolditz**, Katrin Mang, and Thomas Wick. "A modified combined active-set Newton method for solving phase-field fracture into the monolithic limit". In: *Computer Methods in Applied Mechanics and Engineering* 414 (2023), p. 116170.

[4] **Leon Kolditz**, Samy Dray, Viktor Kosin, Amélie Fau, François Hild, and Thomas Wick. "Employing Williams' series for the identification of fracture mechanics parameters from phase-field simulations". In: *Engineering Fracture Mechanics* 307 (2024), p. 110298.

[5] **Leon Kolditz** and Thomas Wick. "Matrix-Free Geometric Multigrid Preconditioning of Combined Newton-GMRES for Solving Phase-Field Fracture with Local Mesh Refinement". In: *Numerical Mathematics and Advanced Applications ENUMATH 2023, Volume 2*. Ed. by Adélia Sequeira, Ana Silvestre, Svilen S. Valtchev, and João Janela. Cham: Springer Nature Switzerland, 2025, pp. 64–74.

[6] Viktor Kosin, Amelie Fau, Francois Hild, and Thomas Wick. "Goal-oriented error estimation for space-time adaptivity in phase field fracture". In: *Journal of computational Physics* (preparation).

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