International Research Training Group IRTG 2657
Computational Mechanics Techniques in High Dimensions

A Hybrid Numerical Methodology Coupling Model Reduction and Graph Neural Networks for Non-parametric 3D Geometries

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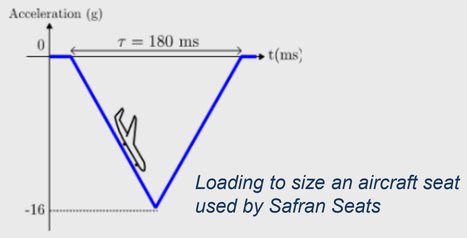
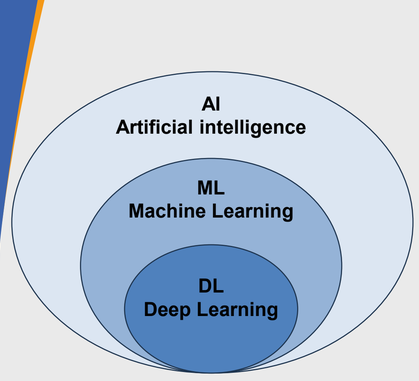
Second Cohort – Starting Date : 08.12.2025, CIFRE Project with Safran Tech

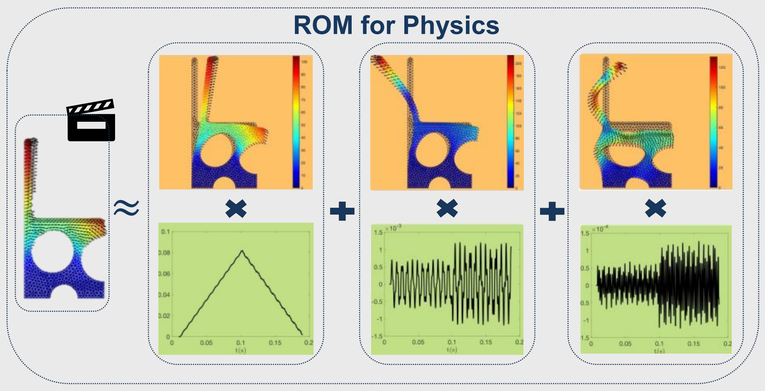
Motivation



Loading to size an aircraft seat used by Safran Seats

There are many innovative design proposals for aircraft seats, but the validation process takes a long time, which causes delays.





AI for speed & generalizability

GOAL: *Develop a tool that quickly pre-sizes new seat designs with new geometry/topology*

Model Order Reduction (MOR)

$$\mathcal{L}(u,\mu) = 0, \forall \mu \in D$$

Governing equation

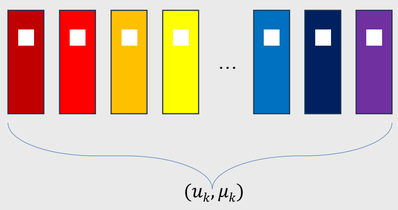
Looking for

$$u(x,t) = \Sigma_i \lambda_i(t) * \Phi_i(x)$$

Offline Phase (long)

Φ_i**?**

Snapshots

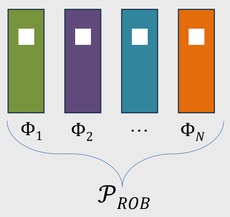


Proper Orthogonal Decomposition (POD)

[Lumley et al., 1967]

[Sirovich, 1987]

Reduced Order Basis



Online Phase (fast)

λ_i**?**

Projection of the PDE :

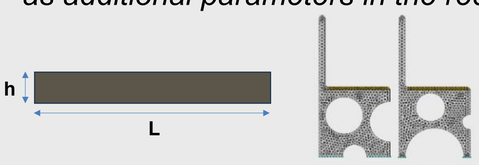
e.g static

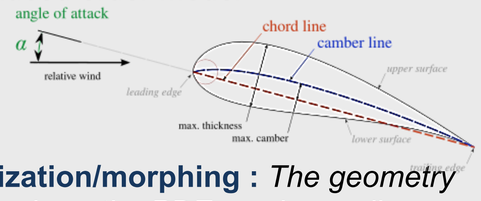
$$\mathcal{P}^T{}_{ROB} \cdot [\mathbb{K}\mathcal{P}_{ROB}\lambda(t) = f(t)]$$

Standard MOR Limitations

Geometric Variations

Geometry is parametric : *The geometry variations can be included as additional parameters in the reduced order model*

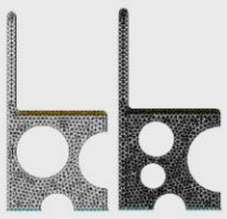




Unknown but existing parametrization/morphing : *The geometry is mapped to a baseline geometry where the PDE can be easily solved using ROM techniques* [Casenave et al., 2023]

Geometry is non-parametric and morphing doesn't exist: *large geometric variations, change of topology …*

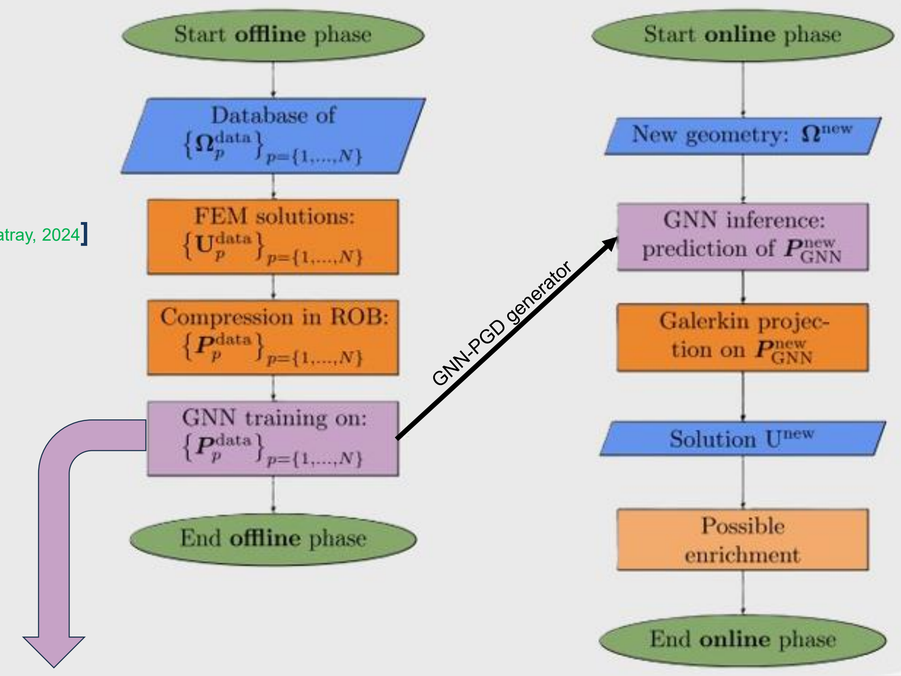
MOR techniques cannot exploit previous geometries computations to speed up those of a new topology



GNN-PGD

A flowchart diagram introducing the steps of the proposed GNN-PGD methodology (Graph Neural Network - Proper Generalized Decomposition)

[Matray, 2024]



Algorithm 1: GNN-PGD description

Require: x_i: input feature vector of node i; A: graph adjacency matrix
Ensure: p_i: output feature vector of node i

1: **Encode:** $h_i^0 \leftarrow \text{MLP}_{\text{node\_encoder}}(x_i)$
2: **Process:**
3: **for** $\ell = 0$ to $L-1$ **do**
4:

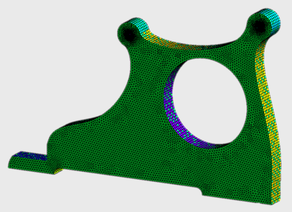
$$h_i^{(\ell+1)} = \phi\left(h_i^\ell, \bigoplus_{j \in \mathcal{N}(i)} \psi\left(h_i^\ell, h_j^\ell, x_i - x_j\right)\right)$$

5: Instance Normalization of $h_i^{(\ell+1)}$
6: **end for**
7: **Decode:** $p_i \leftarrow \text{MLP}_{\text{node\_decoder}}(h_i^L)$
8: **return** p_i

Function	Inputs size	Outputs size	Details	Learnable?
$\text{MLP}_{\text{node\_encoder}}(\bullet)$	k	H	linear ; SiLU ; linear ; SiLU ; LayerNorm	✓
$\psi^\ell(\bullet)$	H	H	linear ; SiLU ; linear ; SiLU	✓
$\bigoplus(\bullet)$	H	H	mean function	×
$\phi^\ell(\bullet)$	H	H	linear ; SiLU ; linear ; SiLU	✓
$\text{MLP}_{\text{decoder}}(\bullet)$	H	q	linear ; SiLU ; linear	✓

Description of the architectures of the various MLPs employed to build the GNN-PGD algorithm

Results



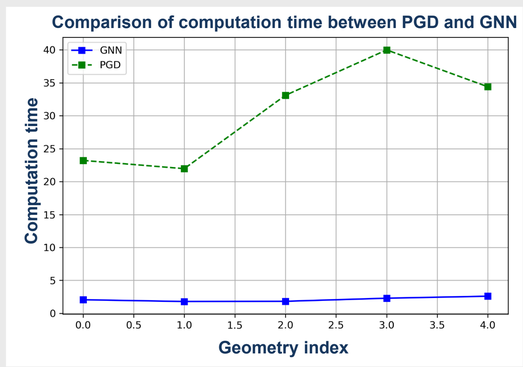
Studied geometry with different topologies (one hole, two holes, ...)

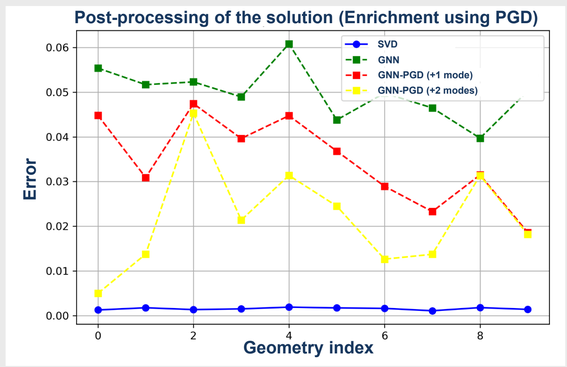
GOAL : *Prediction of the first three PGD modes*

Mode	RMSE	Q^2	Relative RMSE
1	4.3165×10^{-4}	0.99615	5.79%
2	1.0604×10^{-3}	0.96555	14.15%
3	1.7772×10^{-3}	0.90037	23.59%

RMSE : Root Mean Square Error

Current best results – GNN optimization ongoing





These are the results achieved during my research internship prior to my thesis

Next Steps

- **September 2025 – December 2025:** *Optimization and refinement of the developed Graph Neural Network (GNN) model.*
- **January 2026 – December 2026:** *Extension and application of the optimized model to nonlinear behaviors.*
- **January 2027 – December 2027:** *Validation of the approach through applications to real-world case studies and practical scenarios.*

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Funded by DFG Deutsche Forschungsgemeinschaft German Research Foundation