International Research Training Group IRTG 2657
Computational Mechanics Techniques in High Dimensions

Efficient Bridging of Temporal Scales for Fatigue Simulations via Compressive Sensing Approach

Jannik Jarms, David Néron, Udo Nackenhorst
Second Cohort – Starting Date: 01.04.2025

Project Overview

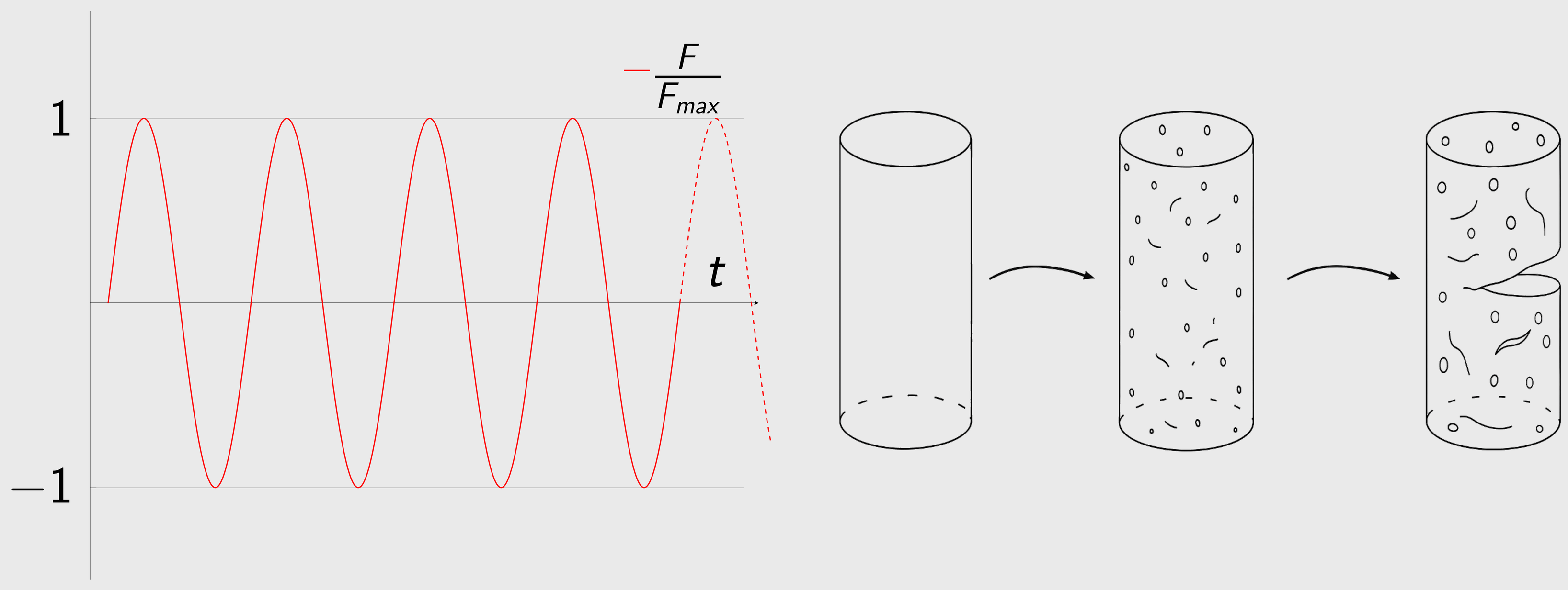
Challenges in Fatigue Damage Simulations:

- High-cycle fatigue: up to 10^9 load cycles are investigated
- Non-linear system of equations for each time instance
- Shannon-Nyquist sampling theorem prescribes treatment of damage evolution for cyclic load sequence:

$$f_{sample} \geq 2 f_{load,max} \tag{1}$$

Project Research Objective:

- Model Order Reduction in space & time dimensions
- Non-intrusive POD-RB methods for ROM in space
- Compressive Sensing: Reduce required number of simulated load cycles significantly assuming sparse representation in suitable basis



(a) Cyclic Load Sequence (b) Schematic Damage Evolution

Figure: Damage induced by cyclic loading.

Gradient Enhanced Damage Model

Framework: Continuum Damage Mechanics, Finite Element Method

Increased robustness of the damage model by introduction of **gradient enhanced free energy function**, ϕ as additional nodal DOF:

$$\Psi(\varepsilon^e, \phi, \nabla\phi, D) = \rho\psi = \frac{1}{2}\varepsilon^e : (1-D)\mathbb{C}^e : \varepsilon^e + \underbrace{\frac{\alpha}{2}(\phi - D)^2}_{coupling} + \underbrace{\frac{\beta}{2}|\nabla\phi|^2}_{penalty} \tag{2}$$

- Prevention of damage localization and mesh dependence
- Improved robustness to stochastic fluctuations, foundation for introduction of randomness and stochastic analysis
- Increased numerical stability, improved convergence behavior

Resulting **variational problem** statements:

$$\int_\Omega \frac{\partial \Psi}{\partial \varepsilon} : \delta\varepsilon \, dV - \int_\Omega b \cdot \delta u \, dV - \int_{\partial\Omega} t \cdot \delta u \, dA \stackrel{!}{=} 0 \quad \forall\, \delta u \tag{3}$$

$$\int_\Omega \alpha(\phi - D)\delta\phi \; + \int_\Omega \beta \nabla\phi\nabla\delta\phi \, dV \stackrel{!}{=} 0 \quad \forall\, \delta\phi \tag{4}$$

Compressive Sensing

Recovery of time trajectories $y(t)$ from $m < n$ measurements y with sub-Nyquist sampling rates results in an underdetermined system equations:

$$\mathbf{y} = \mathbf{A}\mathbf{x_0} = \mathbf{\Phi}\mathbf{D}\mathbf{x_0} \tag{5}$$

Assumption of sparsity of the vector $\mathbf{x_0}$ with n coefficients enables reconstruction of $\mathbf{x} \approx \mathbf{x_0}$ via constrained non-convex, NP-hard ℓ_0-norm optimization problem:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad \text{subject to} \quad \mathbf{y} = \mathbf{A}\mathbf{x} \tag{6}$$

ℓ_1-norm minimization as convex surrogate problem:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{y} = \mathbf{A}\mathbf{x} \tag{7}$$

Approximate solution of ℓ_1-norm minimization through Least Absolute Shrinkage Selection Operator (LASSO):

$$\min_{\mathbf{x}} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1 \tag{8}$$

- **Solution techniques**: Gradient Descent Method, Proximal Gradient Method, Least Angle Regression, ISTA, FISTA, [...]
- **Engineering applications**: Signal reconstruction, structural health monitoring, damage detection, approximation of stochastic PDEs, UQ

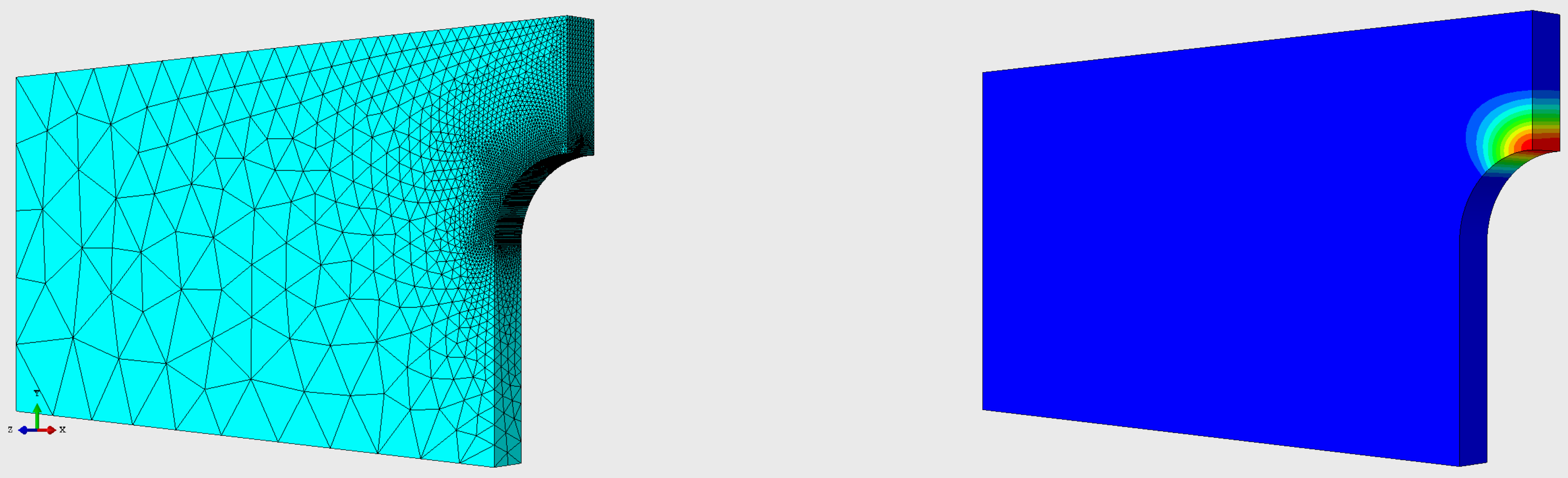
First Semester Summary

Literature Review:

- Finite Element Theory, Damage Modelling, Continuum Damage Mechanics, Stochastic FEM techniques
- Model Order Reduction Techniques, Compressive Sensing

Implementation of Gradient Enhanced Damage Model:

- Commercial *FEM* Software: *Simulia Abaqus*
- User Element Subroutine for 4-node tetrahedral elements



(a) Mesh (b) Nodal Damage Distribution ϕ

Figure: Benchmark Specimen for Damage Model

Next Steps

Sparse Polynomial Chaos Expansion

- Identification of PCE coefficients as sparse recovery problem
- Establish deeper understanding of Compressive Sensing principles
- Explore limitations, potentials and performance of available techniques

Construction of Robust Reduced Order Model in Space

- MOR in space for single load cycles to reduce computational effort
- Proper Orthogonal Decomposition in combination with Radial Basis Functions to circumvent requirement of affine decomposability of the system matrices

Compressive Sensing for Model Order Reduction in Time

- Simulation of significantly reduced number of load cycles than prescribed by Shannon-Nyquist theorem
- Reconstruction of damage evolution from sparse information in time

IRTG Environment

- **Shared Office Space:** Open and welcoming atmosphere, interdisciplinary exchange and discussions on a regular basis
- **Project Supervision:** Open door policy, direct, flexible and reliable
- **GAMM Summer School 2025** in Dresden - Model Order Reduction
- **Stay ENS Paris-Saclay:** January - June 2027

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