International Research Training Group IRTG 2657
Computational Mechanics Techniques in High Dimensions

Modeling of Biofilms in Flow Fields using Porous Medium Theory and the One-Domain Approach

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Motivation

- Biofilms are communities of microorganisms encased in a self-produced protective matrix.
- They present significant challenges in medicine and industry:
  - **Health:** Associated with persistent infections like cystic fibrosis and dental plaque.
  - **Industry:** Cause microbiologically influenced corrosion (MIC) through the production of hydrogen sulfide or organic acids.
- Modeling biofilms involves complex multi-physics, coupling biological, chemical, and mechanical processes.
- **This work focuses on:** Modeling the mechanical deformation and detachment of biofilms in a surrounding flow field.

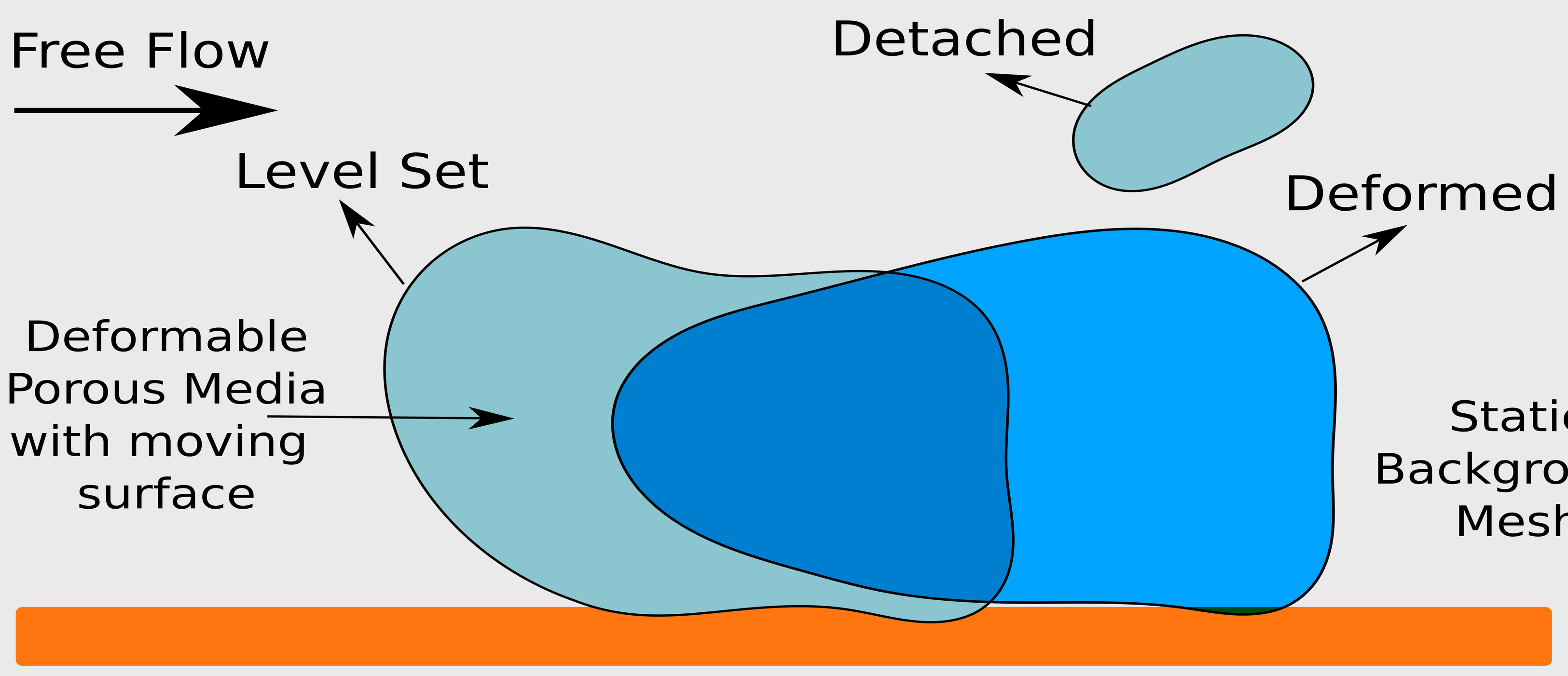

Figure: Biofilm under a flow-field

Model

x: Spatial coordinate	n: Eulerian porosity	$\vec{v}^f$: Fluid velocity
t: Time	$\vec{v}^s$: Solid velocity	$\vec{v}^w$: Pore fluid velocity
$\vec{v}^{ws}$: Relative velocity	p^f: Free fluid pressure	p^w: Pore fluid pressure
p^s: Solid phase pressure	ρ^w: Pore fluid density	ρ^s: Solid density
μ^w: Fluid viscosity	$\mathcal{K}$: Intrinsic permeability	$\boldsymbol{\sigma}^f$: Free fluid stress
$\boldsymbol{\sigma}$: Total Cauchy stress	$\vec{g}$: Gravity acceleration	α: Biot coefficient
K^s: Solid bulk modulus	α_{BJ}: Beavers–Joseph slip coefficient	

Let $\Omega = \Omega_f \cup \Omega_p \subset \mathbb{R}^d$ $(d = 2, 3)$ with interface $\Gamma_{fp} := \partial\Omega_f \cap \partial\Omega_p$. The unit normal on Γ_{fp}, pointing from Ω_p towards Ω_f, is $\boldsymbol{n}$. Determine

$$(\vec{v}^f, p^f) \text{ in } \Omega_f \times I, \qquad (\vec{v}^s, \vec{v}^w, p^s, p^w) \text{ in } \Omega_p \times I.$$

for the isothermal process.

- **Physics in Free flow**

$$\rho^w\big(\partial_t \vec{v}^f + (\vec{v}^f \cdot \nabla)\vec{v}^f\big) = \operatorname{div}\left(\boldsymbol{\sigma}^f\right) + \rho^w \vec{g},$$
$$\operatorname{div}\left(\vec{v}^f\right) = 0,$$
$$\boldsymbol{\sigma}^f = -p^f\ \mathbf{I} + 2\mu^w \operatorname{sym}\operatorname{grad}\left(\vec{v}^f\right).$$

- **Coupled Physics in Porous medium**
  - **Momentum balance for solid**

$$\begin{aligned}\rho^s(1-n)\,\partial_t\vec{v}^s + \rho^w n\,\partial_t\vec{v}^w + \operatorname{grad}\left(\rho^s(1-n)\tfrac{1}{2}|\vec{v}^s|^2\right) + \operatorname{grad}\left(\rho^w n\tfrac{1}{2}|\vec{v}^w|^2\right)\\ - \tfrac{1}{2}|\vec{v}^w|^2 \operatorname{grad}(\rho^w n) - \tfrac{1}{2}|\vec{v}^s|^2 \operatorname{grad}(\rho^s(1-n))\\ - \rho^s(1-n)\vec{v}^s \times \operatorname{curl}(\vec{v}^s) - \rho^w n \vec{v}^w \times \operatorname{curl}(\vec{v}^w) = \rho\vec{g} + \operatorname{div}(\boldsymbol{\sigma}),\end{aligned}$$

  - **Momentum balance for fluid**

$$\begin{aligned}n\,\vec{v}^{ws}\frac{\mu^w}{\mathcal{K}} = -\operatorname{grad}(p^w) + \rho^w\vec{g} - \rho^w\,\partial_t\vec{v}^w - \rho^w\frac{1}{2}\Big(\operatorname{grad}(\vec{v}^s\cdot\vec{v}^w) - \vec{v}^s \times \operatorname{curl}(\vec{v}^w)\\ - \vec{v}^w \times \operatorname{curl}(\vec{v}^s) - \operatorname{curl}(\vec{v}^s \times \vec{v}^w) + \vec{v}^s\operatorname{div}(\vec{v}^w) - \vec{v}^w\operatorname{div}(\vec{v}^s)\Big),\end{aligned}$$

  - **Mass balance for solid**

$$\begin{aligned}\frac{\alpha-n}{K^s}\partial_t p^s - \partial_t n + \operatorname{div}\left(p^s\frac{\alpha-n}{K^s}\vec{v}^s\right) - p^s\operatorname{div}\left(\tfrac{\alpha-n}{K^s}\vec{v}^s\right)\\ + \operatorname{div}((\alpha-n)\vec{v}^s) - \vec{v}^s\cdot\operatorname{grad}(\alpha-n) - \operatorname{div}(n\,\vec{v}^s) + n\operatorname{div}(\vec{v}^s) = 0.\end{aligned}$$

  - **Mass balance for fluid**

$$\begin{aligned}\left(\frac{\alpha-n}{K_s} + \frac{n}{K_w}\right)\frac{\partial p^w}{\partial t} + \alpha\operatorname{div}(\vec{v}^s) + \operatorname{div}(n\vec{v}^{ws}) + \frac{\alpha}{K_s}\operatorname{div}(p^w\vec{v}^s) - \frac{\alpha}{K_s}p^w\operatorname{div}(\vec{v}^s)\\ -\frac{1}{K_s}n\operatorname{div}(p^w\vec{v}^s) + \frac{1}{K_s}np^w\operatorname{div}(\vec{v}^s) + \frac{1}{K_w}n\operatorname{div}(p^w\vec{v}^w) - \frac{1}{K_w}np^w\operatorname{div}(\vec{v}^w) = 0\end{aligned}$$

  - **Interface conditions**

$$\vec{v}^f\cdot\boldsymbol{n} = \vec{v}^s\cdot\boldsymbol{n} \qquad , \boldsymbol{\sigma}^f\boldsymbol{n} = \boldsymbol{\sigma}\boldsymbol{n},$$
$$\vec{v}^w\cdot\boldsymbol{n} = -(\vec{v}^f - \vec{v}^s)\cdot\boldsymbol{n}.$$

- **Numerical Solving Method**
- **Goal of the model.** Predict accurate deformation of the poroelastic biofilm under shear.
- **One-domain approach.** Free fluid and biofilm are solved on the same background mesh; the film region is represented implicitly (indicator/level set).
- **Interface capturing.** Unfitted discretization via the *Cut Finite Element Method (CutFEM)* with ghost-penalty stabilization and Nitsche coupling.
- **Acceleration.** Planned **MOR** for the poroelastic subproblem to enable long-time or parameter-sweep studies of detachment.

First Verifications

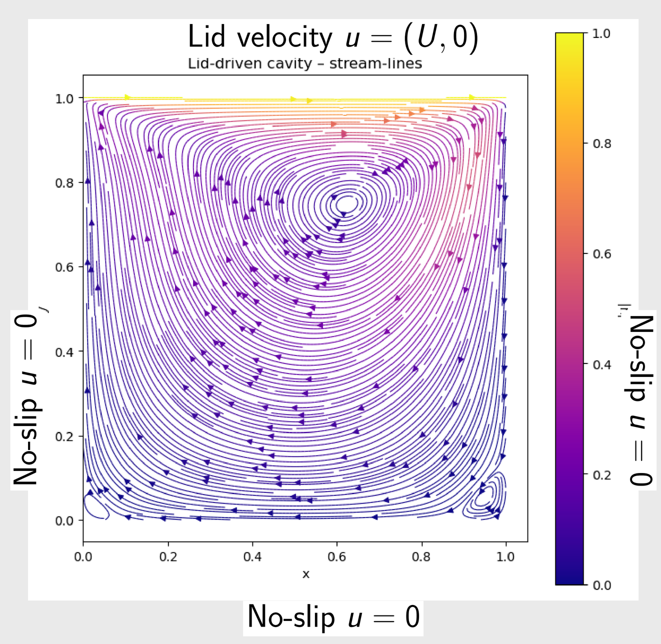

Figure: Lid-driven cavity (2D incompressible): *BCs* — top lid $u = (U, 0)$; other walls no-slip $u = 0$.

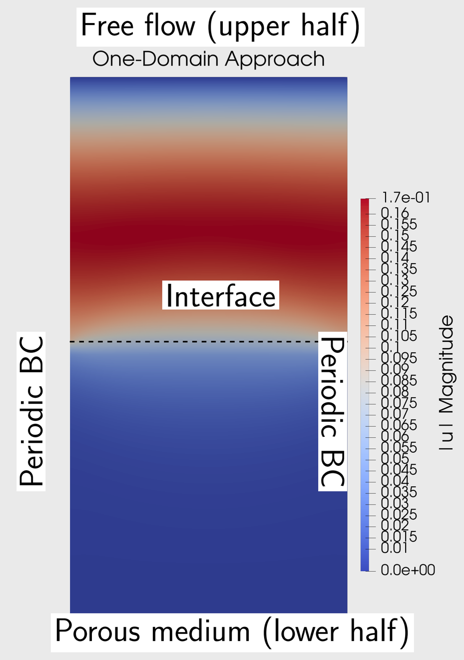

Figure: One-domain setup: upper half = free flow, lower half = porous medium; *BCs* — periodic left/right.

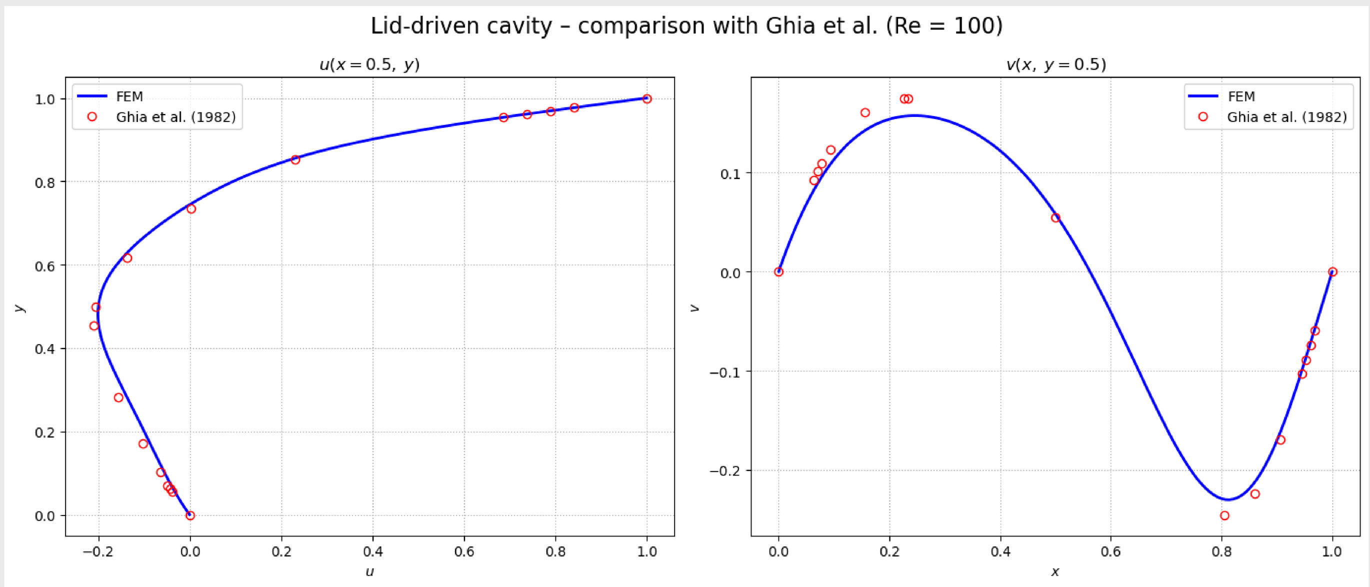

Figure: Comparison with Ghia et al.: centerline velocity profiles for the 2D lid-driven cavity.

Achievements & Outlook

- **Model improvement.** Reformulated the framework around variable porosity/permeability and poroelastic response.
- **Library / discretization.** Developing an extensible *Cut Finite Element Method (CutFEM)* stack:
  - Ghost-penalty stabilization (arbitrary order) and robust cut-cell quadrature,
  - Nitsche-type coupling for fluid/film interfaces,
  - Regression suite includes lid-driven cavity benchamrk and ∼96 additional tests.
- **Roadmap (next).** FSI and FPI comparisons; *large-scale biofilm detachment scenarios and parameter sweeps*; **MOR integration**.
- **Planned verification.** Terzaghi-type consolidation test for the poroelastic block and detachment data (Stoodley et al., 2002).

IRTG Environment and Activities

- **Network & supervision.** Cross-disciplinary collaboration with doctoral researchers in mechanics, chemistry, and numerical analysis; regular co-advisory feedback and weekly *Jour fixe*.
- **Planned mobility (2026).** Research stay at **ENS Paris-Saclay** focused on **model order reduction (MOR)** for poroelastic/biofilm detachment models.

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