International Research Training Group IRTG 2657
Computational Mechanics Techniques in High Dimensions

On a Modular Coupling Strategy for High-Performance Computing of Multiphysics Problems

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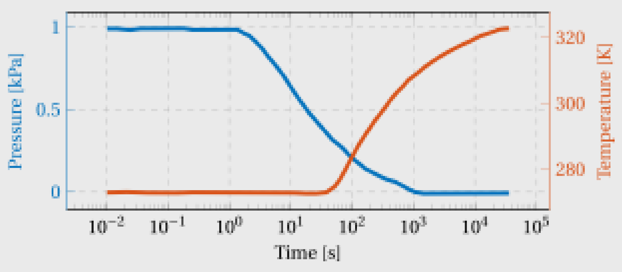
Second Cohort – Starting Date: 01.09.2024

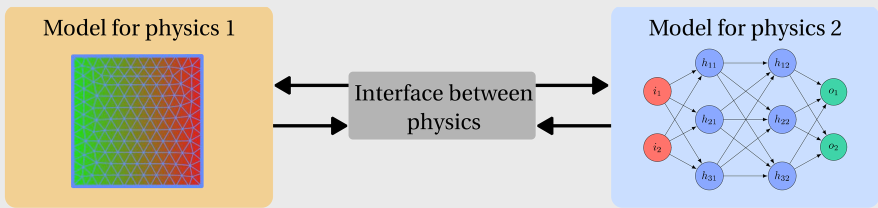
Motivation

- Strong coupling: mutual interaction between different physical phenomena
- Coupling terms leading to **numerical difficulties**
- **Different** numerical methods for each physics

Different dynamics per physics



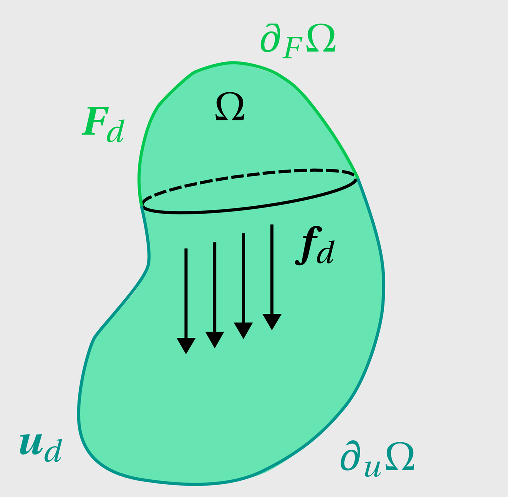
Different models per physics



Need for a suitable and modular coupling strategy
Case of application: nonlinear strongly coupled **thermo-poroelasticity**

Model

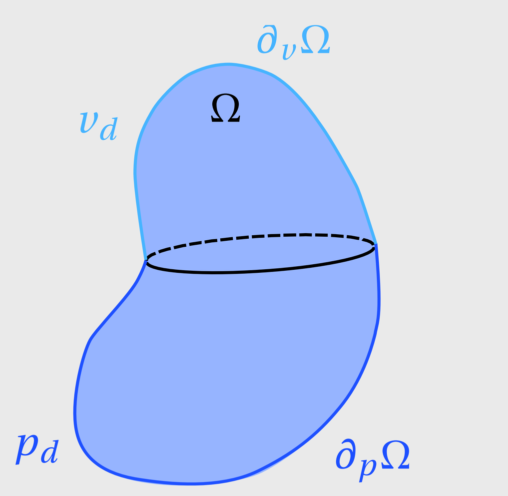
Solid physics admissibility



$$\begin{cases} \boldsymbol{u} = \boldsymbol{u}_d \text{ on } \partial_u\Omega \\ \boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^{\mathrm{T}}\right) \text{ in } \Omega \times I \end{cases}$$

$$\begin{cases} \boldsymbol{\sigma}\boldsymbol{n} = \boldsymbol{F}_d \text{ on } \partial_F\Omega \\ \nabla\cdot\boldsymbol{\sigma} + \boldsymbol{f}_d = \boldsymbol{0} \text{ in } \Omega \times I \end{cases}$$

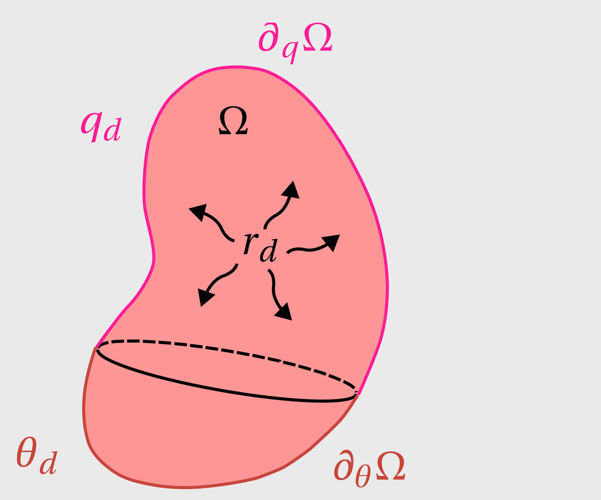
Fluid physics admissibility



$$\begin{cases} p = p_d \text{ on } \partial_p\Omega \\ \boldsymbol{Z} = \nabla p \text{ in } \Omega \times I \end{cases}$$

$$\begin{cases} \boldsymbol{V}\cdot\boldsymbol{n} = v_d \text{ on } \partial_v\Omega \\ -\nabla\cdot\boldsymbol{V} = \xi \text{ in } \Omega \times I \end{cases}$$

Thermal physics admissibility



$$\begin{cases} \theta = \theta_d \text{ on } \partial_\theta\Omega \\ \boldsymbol{X} = \nabla\theta \text{ in } \Omega \times I \end{cases}$$

$$\begin{cases} \boldsymbol{q}\cdot\boldsymbol{n} = q_d \text{ on } \partial_q\Omega \\ -\nabla\cdot\boldsymbol{q} + r_d = R \text{ in } \Omega \times I \end{cases}$$

Coussy, *Poromechanics*, 2004

Constitutive coupling relations

$$\begin{cases} \boldsymbol{\sigma} = \boldsymbol{D}:\boldsymbol{\varepsilon} - b p\boldsymbol{I} - \beta\theta\boldsymbol{I} \\ \boldsymbol{V} = -H\boldsymbol{Z} \\ \xi = \frac{1}{M}\dot{p} + b\mathrm{Tr}\dot{\boldsymbol{\varepsilon}} - 3\alpha_m\dot{\theta} \\ \boldsymbol{q} = -k\boldsymbol{X} \\ R = C_d\dot{\theta} + \beta T_0\mathrm{Tr}\dot{\boldsymbol{\varepsilon}} - 3\alpha_m T_0\dot{p} + \rho_f c_f \boldsymbol{V}\cdot\boldsymbol{X} - \frac{1}{H}\left(1 - 3\alpha_f T_0\right)\boldsymbol{V}^2 \end{cases}$$

Nonlinearities:
- heat convectively transported by the fluid
- viscous dissipation in the fluid

Solid ↔ Fluid coupling

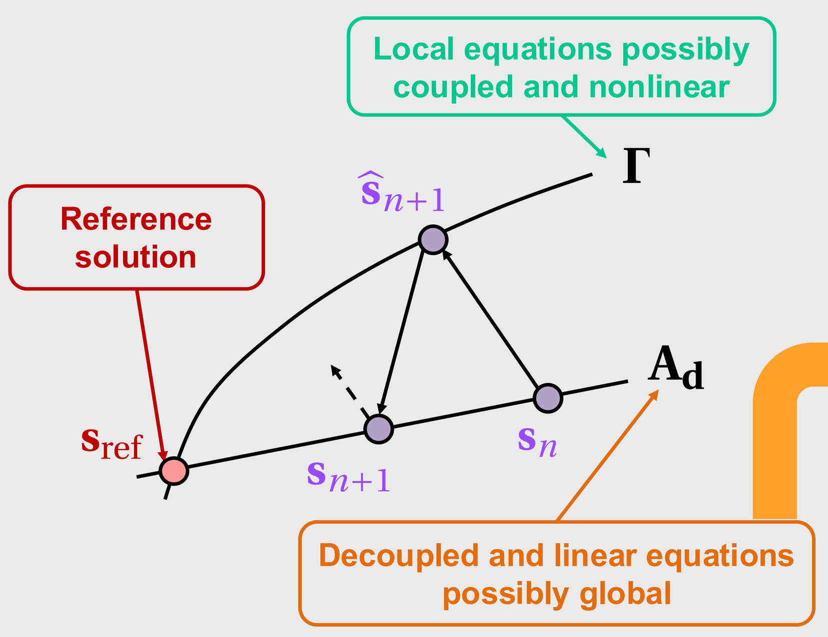
Solid ↔ Thermics coupling

Fluid ↔ Thermics coupling

Method

Main idea:
Separate coupled and decoupled equations

Ladevèze 1999
Dureisseix et al 2003



Decoupled stage solved with ***Proper Generalized Decomposition*** (PGD)

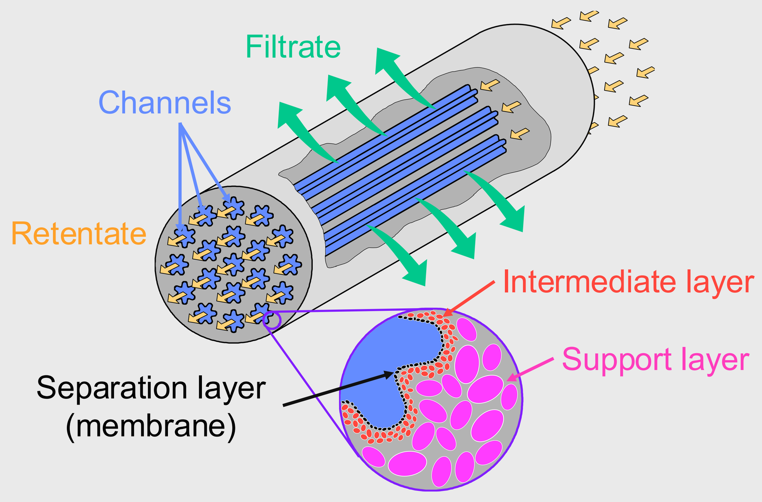
$$\begin{cases} \mathbf{u}_n(t) = \mathbf{u}_0(t) + \sum_{i=1}^{k} \lambda_i(t)\overline{\mathbf{u}}_i & k \leqslant n \\ \mathbf{p}_n(t) = \mathbf{p}_0(t) + \sum_{i=1}^{l} \gamma_i(t)\overline{\mathbf{p}}_i & l \leqslant n \\ \theta_n(t) = \theta_0(t) + \sum_{i=1}^{m} \varphi_i(t)\overline{\theta}_i & m \leqslant n \end{cases}$$

Using the solver to perform several computations with material variability

Boucard, Champaney 2003

1. **Reuse** the solution of the previous computation as an initialization
2. **Enrich the basis** created all along the previous computations

Results

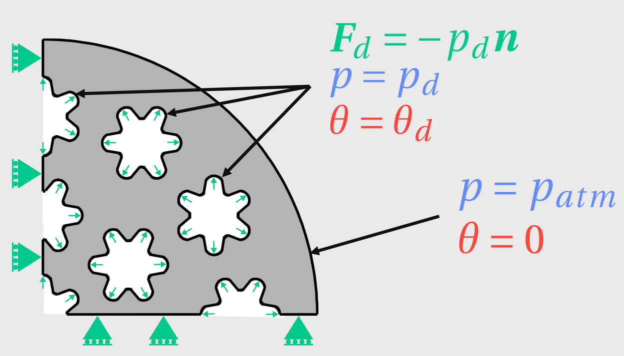


Context:

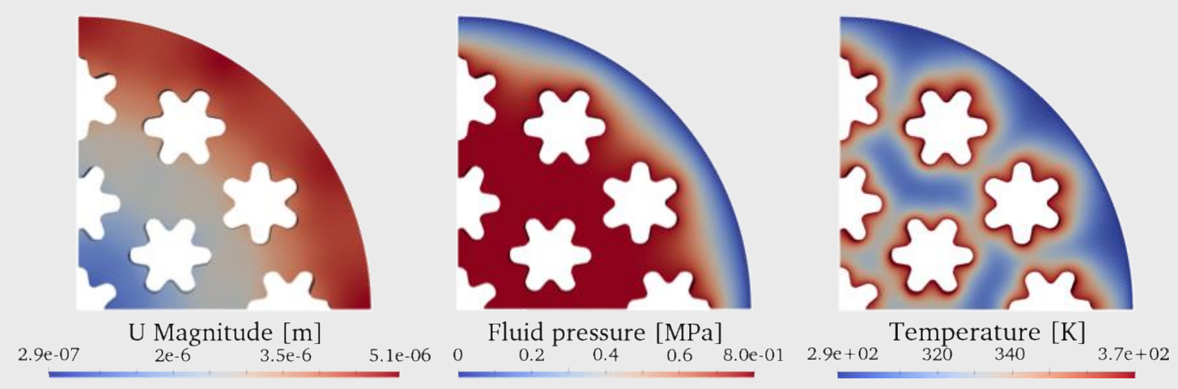
- Ceramic membranes used to **filter fluids**
- Different layers constituted of **porous materials**
- Bacterian retained by the pores when fluid crosses the ceramics
- Membrane developed by $Mantec$ and using the $Star - Sep^{TM}$ technology.

Numerical parameters

- Quarter of the structure studied due to **symmetries**
- 216,364 DoFs
- 120 time steps



Results for a single computation at t = 25 ms



PGD modes:

- Displacement: 15 modes
- Pore pressure: 6 modes
- Temperature: 18 modes

Results for a parametric study with 343 computations

$k_f \in [0.8, 1.2] \cdot k_{f,0}$ with 7 points

$\alpha_s \in [0.8, 1.2] \cdot \alpha_{s,0}$ with 7 points

$H \in [0.8, 1.2] \cdot H_0$ with 7 points

Gain of the method

$$G = \frac{343\tau_{single}}{\tau_{total}} = 20.35$$

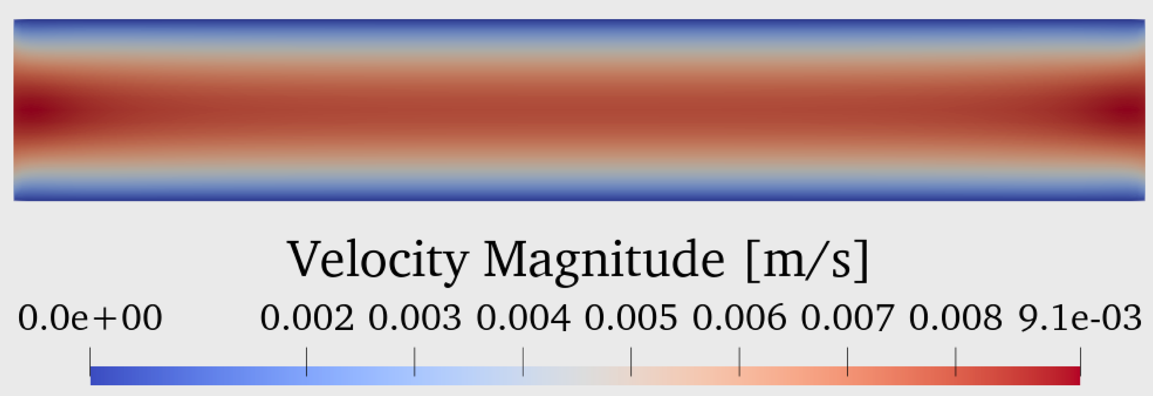
Next Steps

- **Multirate and multimesh approach:**
  - Use different time discretizations and meshes for each physics
- **Multimodel approach:**
  - Coupling data-based model with a finite element one

Research Environment

- Spokesperson doctoral candidates IRTG 2657 (French/Paris-Saclay side)
- Workshop CSMA Juniors 2025, Toulouse, France
- Participation in *Congrès Français de Mécanique 2025*, Metz, France
- Two stays at Leibniz Universität Hannover (summer 2025 + summer 2026)
  - Adapting the LATIN-PGD coupling strategy to non-Newtonian fluids using internal variables

Junker, Wick 2025



First results on laminar Newtonian flows: coupling between velocity and pressure

Publication

[1] **E. Foulatier**, D. Néron, F. Louf, P.-A. Boucard, *A strategy with reduced models dedicated to parametrized nonlinear strongly coupled thermo-poroelasticity problems*, Advanced Modeling and Simulation in Engineering Sciences, 2025, <hal-05219510> (accepted)

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