International Research Training Group IRTG 2657
Computational Mechanics Techniques in High Dimensions

A space-time formulation for thermodynamic topology optimization for inelastic materials

Christian Sellmann, Thomas Wick, David Neron, Philipp Junker
Second Cohort – Doctoral Researcher – Starting Date: 01.09.2024

Motivation

Problem

- Topology optimization of nonlinear time-dependent materials computationally complex
- Material evolution needs to be recalculated in each optimization step

Solution approach

- Utilization of Thermodynamic Topology Optimization (TTO) derived from Hamilton's principle of stationary action

$$\mathcal{H}[\mathbf{u},\alpha,\chi] = \mathcal{G}[\mathbf{u},\alpha,\chi] + \mathcal{D}[\alpha,\chi] - \mathcal{R}[\chi] + \mathcal{C}[\alpha,\chi] \rightarrow \underset{\mathbf{u},\alpha,\chi}{\mathbf{stat}}$$

- Time evolution of internal variables and design variable on one time scale
- Leveraging Space-Time-Method with adaptive mesh refinement for higher computational efficiency

Numerical Methods

Virtual Element Method

- Generalization of FEM for polygonal elements
- Approximation of shape functions inside each element
  - Split in consistency and stabilization part

$$A(u_h,\phi_h) \approx A(u_h^\pi,\phi_h^\pi) + \gamma \sum_{i=1}^{\mathrm{NDOF}} \mathrm{dof}_i(u_h - u_h^\pi)\,\mathrm{dof}_i(\phi_h - \phi_h^\pi)$$

  - Stabilization parameter γ needs to be estimated
- Can handle almost arbitrary element shapes
  - Straightforward handling of hanging nodes during mesh refinement

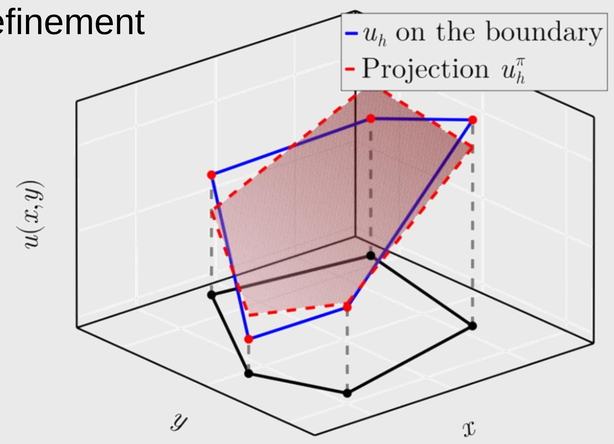

Figure: Concept of the VEM projection

Goal-Oriented Dual Weighted Results Error Estimation (DWR)

- For an efficient and reliable numerical simulation
- Allows to estimate the error for almost arbitrary quantities of interest $J(u)$
- Can share the objective function with the topology optimization

$$J(u) - J(u_h) = F(z) - F(z_h) - A(u_h, z_h - z_h^\pi) \\ - A(u_h - u_h^\pi, z_h - z_h^\pi) + S(u_h - u_h^\pi, z_h - z_h^\pi)\,.$$

Stabilization error

Past, Current & Planned Work

- ✓ Implementation of a fully adaptive VEM code in Julia
- ✓ VEM with goal-oriented DWR **[1]**
- ✓ Application of goal-oriented DWR & VEM for TTO in two dimensions
  C. Sellmann, T. Wick, and P. Junker. **"*A Goal-Oriented Error Estimation Framework for Thermodynamic Topology Optimization Using Adaptive Virtual Elements.*** (In preparation). 2025
- ✓ Development of a novel polygonal FEM (PET-FEM)
  C. Sellmann, P. Wriggers, and P. Junker. **"*Projection Enhanced Triangular Finite Element Method: A Novel FEM Approach for Polygonal Elements*".** (In preparation). 2025
- Extending adaptive TTO to 3 dimensions (*ongoing*)
  - Extending PET-FEM to 3 dimensions
  - Space-Time TTO in 2+1 dimensions

Results

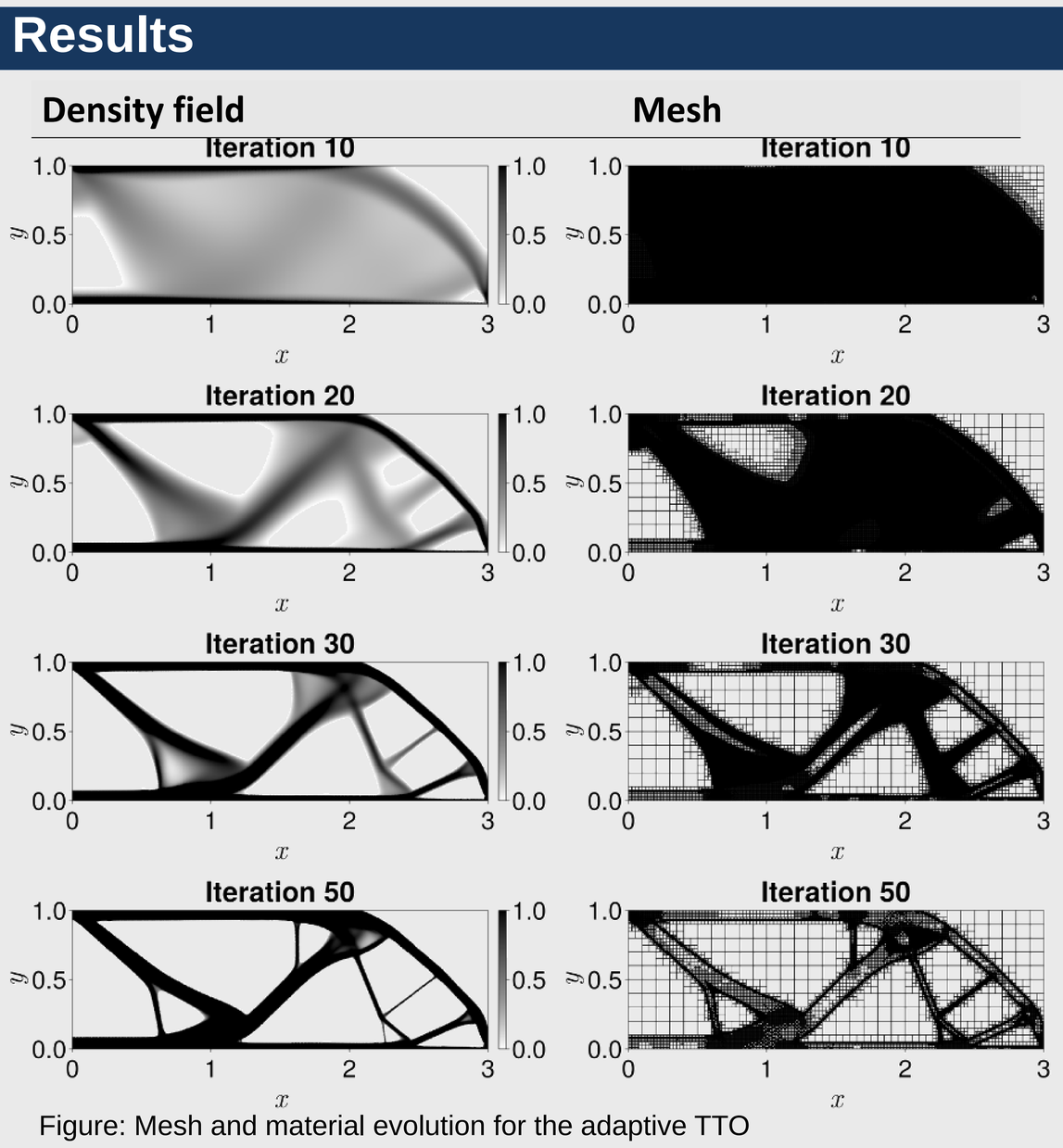

Figure: Mesh and material evolution for the adaptive TTO

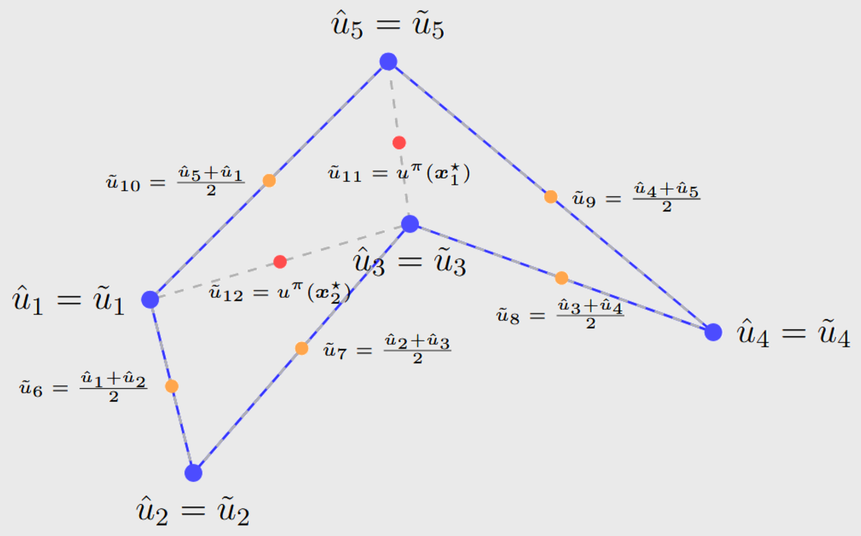

Figure: Concept of the PET-FEM, a VEM alternative

Publications

[1] **C. Sellmann** and P. Junker. **"Goal-Oriented Dual-Weighted Residual Error Estimation for the Virtual Elements Method."** *Computer Methods in Applied Mechanics and Engineering* 443 (August 2025): 118034. https://doi.org/10.1016/j.cma.2025.118034

IRTG Enviroment

- President of the "GAMM Nachwuchsgruppe Hannover"
- Participation in **Conferences, Summer Schools,** and **Workshops**
  - GAMM 2025
  - GACM 2025
  - Course on "Emerging discretization methods in solid mechanics" in Udine 2025
- **Stay at ENS Paris-Saclay**: January 2026 – June 2026

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