International Research Training Group IRTG 2657
Computational Mechanics Techniques in High Dimensions

# Fast Solvers for Monolithic Fluid-Structure Interaction with Space-Time Adaptivity and Model Order Reduction


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## Project Overview

In this project, we want to develop **efficient computational strategies** to address **computationally expensive multi-query optimization for complex fluid structure interaction** multiphysics problems.

- **Fast solvers** for the discrete monolithic variational formulation of fluid structure interaction.
- Devising **effective model-order reduction** strategy
- **Space-time adaptive mesh refinement** using a posteriori goal-oriented error-estimation technique (collobration - Viktor Kosin 1st cohort).

## Numerical Simulation

### Monolithic Formulation

Solid: Lagrangian coordinates    Fluid: Eulerian coordinates

- Handling of distinct coordinate systems in a monolithic fashion – Arbitrary Lagrangian Eulerian (ALE)

### Discretization

- Space – standard Galerkin FEM
- Time – Finite difference one-step $\theta$-scheme

### Linearization

- Newton linearization of non-linear algebraic systems

$$\mathcal{J}(\mathcal{X}^i)\,\Delta\mathcal{X} = -\mathcal{F}(\mathcal{X}^i)$$
$$\mathcal{X}^{i+1} = \mathcal{X}^i + \Delta\mathcal{X}$$

$$\mathcal{J} = \left[\begin{array}{c|cc} \mathbf{S} & \mathbf{C}_{su} & \mathbf{C}_{sp} \\ \hline \mathbf{C}_{us} & \mathbf{F} & \mathbf{G} \\ \mathbf{C}_{ps} & \mathbf{D} & \mathbf{0} \end{array}\right]$$

**(1,1)** Tangent stiffness matrix for the structure

**(2,2)** Discretization of the non-stationary Navier–Stokes equations.

**(2,1)** Fluid force effect on structure

**(1,2)** Structural displacement effect on fluid flow

### Linear Solvers

- Challenges:
  - Monolithic system matrix is large and ill-conditioned
  - Memory bottleneck
- Solutions
  - Krylov subspace-based iterative solvers.
  - Block preconditioners with *Geometric multigrid ingredient* based on LDU factorization of the Jacobian matrix.
  - *Matrix-free approach*
    Iterative solvers dominated by matrix-vector product kernels
    Matrix-vector product computed as:

$$\mathbf{v} = \bigwedge_{\text{cell}=1}^{N_{\text{cells}}} \mathbf{A}_{\text{cell}}\,\mathbf{u}_{\text{cell}}$$

## Progress and Outlook

- Fast solvers
  - Matrix-free based block preconditioned iterative solver for the stationary Stokes problem
  - Extension to the non-stationary Navier-Stokes problem
  - Extension to the fluid structure interaction
- Implementation of model order reduction.
- Space-time adaptivity based on posteriori error estimation.

## IRTG Environment

- Responsible for IRTG 2657 website management
- Colloboration initiative with colleagues (Chiara and Jamal) working on Biofilm modelling
- Planned Stay ENS Paris-Saclay: February 2026 - July 2026.
- colloboration with Viktor Kosin (1st cohort - Doctoral researcher) who has been working on space-time adaptivity for phase-field fracture simulations.

## Model Problem

Let $\Omega_f(t)$ denote the fluid domain and $\Omega_s(t)$ the structure domain, with $\mathcal{I}(t)$ the interface. The total domain is $\Omega(t) = \Omega_f(t) \cup \Omega_s(t) \cup \mathcal{I}(t)$.

### Fluid Problem

Find the fluid velocity $\mathbf{v}_f$ and pressure $p_f$ such that in $\Omega_f(t)$:

$$\rho_f\left(\frac{\partial \mathbf{v}_f}{\partial t} + (\mathbf{v}_f \cdot \nabla)\mathbf{v}_f\right) - \nabla \cdot \boldsymbol{\sigma}_f = \mathbf{f}_f,$$
$$\nabla \cdot \mathbf{v}_f = 0,$$

where $\boldsymbol{\sigma}_f = -p_f \mathbf{I} + \mu_f(\nabla \mathbf{v}_f + \nabla \mathbf{v}_f^T)$ with $\mu_f$, dynamic viscosity of the fluid

### Structural problem

Find the structural displacement $\mathbf{u}_s$ such that in $\Omega_s(t)$:

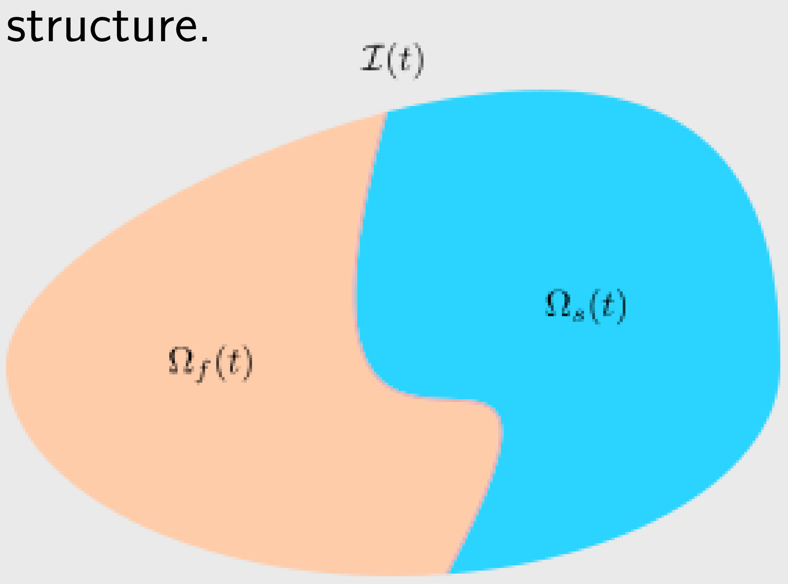
$$\rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2} - \nabla \cdot \boldsymbol{\sigma}_s = \mathbf{f}_s,$$

where $\boldsymbol{\sigma}_s$ is the Cauchy stress tensor

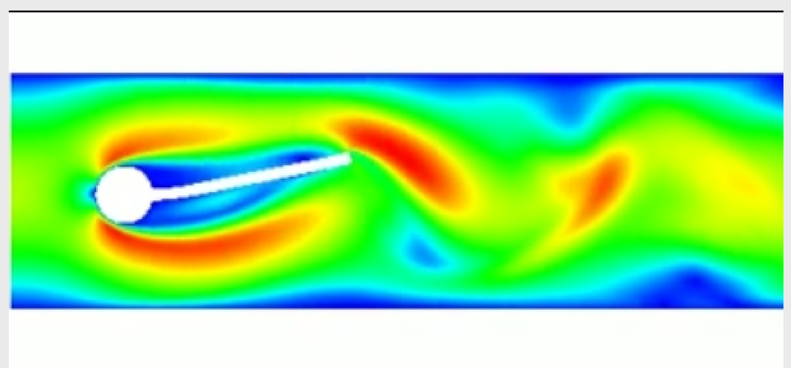
### Coupling Conditions on $\mathcal{I}(t)$

$$\mathbf{v}_f = \frac{\partial \mathbf{u}_s}{\partial t} \qquad \text{(kinematic)},$$
$$\boldsymbol{\sigma}_f \mathbf{n}_f = \sigma \mathbf{n}_s \qquad \text{(dynamic)}.$$

with appropriate boundary and Initial Conditions for both the fluid and the structure.




Domain $\Omega(t)$



Turek and Hron FSI-2 benchmark - velocity field at maximum upward beam deflection [Wick 2013]

## Results

### Stokes flow - step-56 Dealii - simulating convection in the earth mantle

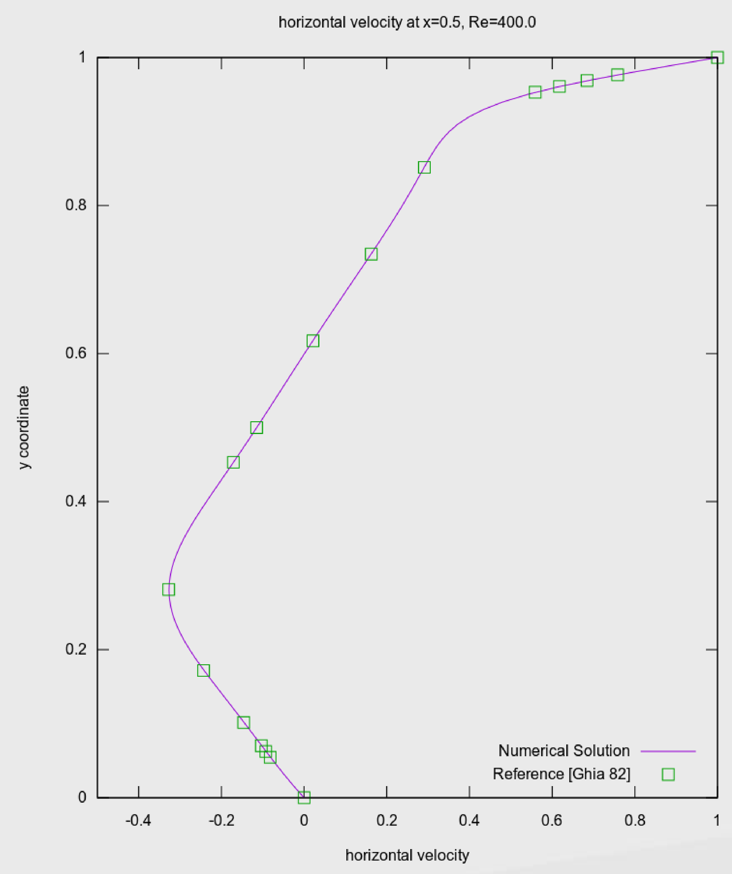
| Cycle | DoFs | Total Time | | Speedup |
|---|---|---|---|---|
| | | Matrix-based | Matrix-free | |
| 0 | 15,468 | 64.871s | 7.835s | *8.3* |
| 1 | 112,724 | 503.42s | 63.968s | *7.9* |
| 2 | 859,812 | - | 480.98s | - |

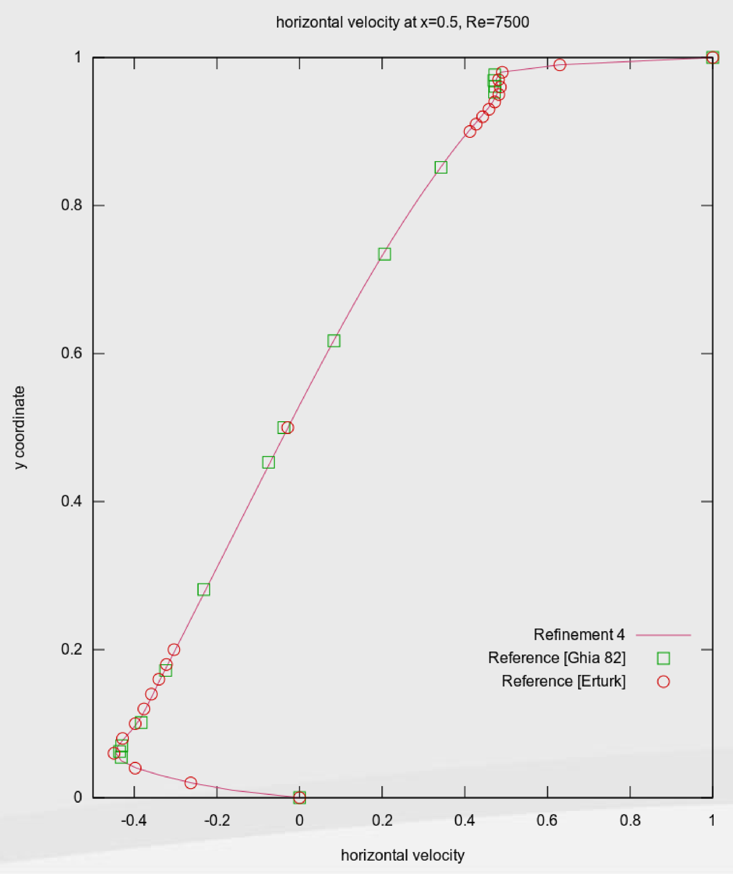
Table: Solver performance comparison: Matrix-Based vs Matrix-Free (timings in seconds)

| Refinement Level | L2 Velocity | Reduction | L2 Pressure | Reduction |
|---|---|---|---|---|
| 3 global refinements | 6.71e-04 | - | 3.65e-03 | - |
| 4 global refinements | 8.38e-05 | 8.0 | 8.85e-04 | 4.1 |
| 5 global refinements | 1.04e-05 | 8.0 | 2.20e-04 | 4.0 |

Table: Convergence Analysis - Matrix-Free Solver

### Stationary Navier-Stokes - Lid driven cavity for Re = 400, 7500





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